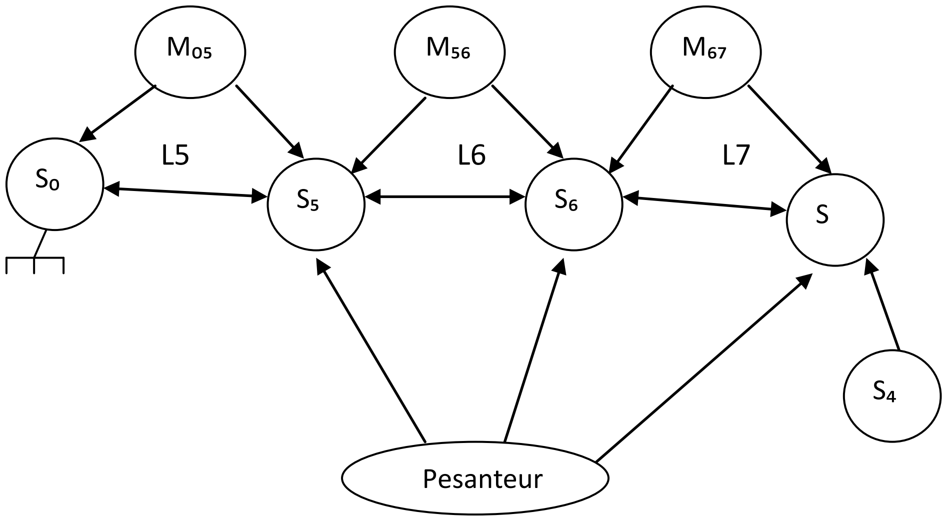
Correction de CNC TSI 2010

Génie mécanique

I. Étude du système porte utile :

1. *Etude statique :*

1.1.1 Schéma d'analyse :



L5 : glissière d'axe $(O_5, \vec{x}_0)$

L6 : glissière d'axe $(O_6, \vec{x}_0)$

L7 : pivot d'axe $(O_7, \vec{x}_0)$

1.1.2 les torseurs :

$$\left\{\tau_{S_0 \to S_5}\right\} = \begin{Bmatrix} 0 \mid L_{05} \\ Y_{05} \mid M_{05} \\ Z_{05} \mid N_{05} \end{Bmatrix}_{(O_5, B_0)} \left\{\tau_{S_5 \to S_6}\right\} = \begin{Bmatrix} X_{56} \mid L_{56} \\ Y_{56} \mid M_{56} \\ 0 \mid N_{56} \end{Bmatrix}_{(O_6, B_0)}$$

$$\left\{\tau_{S_6 \to S_7}\right\} = \begin{Bmatrix} X_{67} \mid 0 \\ Y_{67} \mid M_{67} \\ Z_{67} \mid N_{67} \end{Bmatrix}_{(O_7, B_0)}$$

Rq : le dernier torseur est valable dans toute base qui contient $\vec{x}_0$

1.1.3

$$\left\{\tau_{M_{67}\to S}\right\} = \begin{Bmatrix} 0 \mid C_{67} \\ 0 \mid 0 \\ 0 \mid 0 \end{Bmatrix}_{(O_7,B_0)} \left\{\tau_{pes\to S}\right\} = \begin{Bmatrix} 0 \mid 0 \\ 0 \mid 0 \\ -m.g \mid 0 \end{Bmatrix}_{(O_7,B_0)}$$

$$\left\{\tau_{S_6\to S_7}\right\} = \begin{Bmatrix} X_{67} \mid 0 \\ Y_{67} \mid M_{67} \\ Z_{67} \mid N_{67} \end{Bmatrix}_{(O_7,B_0)} \left\{\tau_{S4\to S}\right\} = \begin{Bmatrix} X_{48} \mid 0 \\ Y_{48} \mid 0 \\ Z_{48} \mid 0 \end{Bmatrix}_{(M,B_0)}$$

- Réduction des torseurs au point O_{07} :

$$\overrightarrow{M}_{O_7} = \overrightarrow{M}_{M(S_4\to S)} + \overrightarrow{O_7M} \wedge \overrightarrow{R}_{S_4\to S} ; \overrightarrow{O_7M} = \begin{pmatrix} e \\ R \\ 0 \end{pmatrix}_{(O_7,B_7)} = \begin{pmatrix} e \\ R\cos\theta_{76} \\ R\sin\theta_{76} \end{pmatrix}_{(O_7,B_0)}$$

$$\overrightarrow{M}_{O_7} = \begin{pmatrix} Z_{48}.R\cos\theta_{76} - Y_{48}R\sin\theta_{76} \\ X_{48}R\sin\theta_{76} - eZ_{48} \\ eY_{48} - X_{48}R\cos\theta_{76} \end{pmatrix}_{(O_7,B_0)} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}_{(O_7,B_0)}$$

- PFS :

$$\left\{\tau_{\bar{S}\to S}\right\}_{O_7} = \begin{Bmatrix} \vec{0} \\ \vec{0} \end{Bmatrix}_{O_7} \Rightarrow \begin{cases} X_{67} + X_{48} = 0 \\ Y_{67} + Y_{48} = 0 \\ Z_{67} + Z_{48} - mg = 0 \end{cases} ; \begin{cases} C_{67} + \alpha = 0 \\ M_{67} + \beta = 0 \\ N_{67} + \gamma = 0 \end{cases}$$

- Equation du mouvement : $C_{67}+\alpha=0$

2. *Etude cinématique :*

1.2.1 Torseurs cinématiques :

$$\left\{\vartheta_{S_5/S_0}\right\}_{O_5} = \begin{Bmatrix} \overrightarrow{0} \\ \dot{\mu}\overrightarrow{x_0} \end{Bmatrix}_{O_5} ; \left\{\vartheta_{S_6/S_5}\right\}_{O_6} = \begin{Bmatrix} \overrightarrow{0} \\ \dot{\delta}\overrightarrow{z_0} \end{Bmatrix}_{O_6} ; \left\{\vartheta_{S/S_6}\right\}_{O_7} = \begin{Bmatrix} \dot{\theta}_{76}\overrightarrow{x_0} \\ \vec{0} \end{Bmatrix}_{O_7}$$

1.2.2

$$\overrightarrow{V_{M\in S/R_0}} = \left.\frac{d\overrightarrow{O_0M}}{dt}\right|_{R_0} - \left.\frac{d\overrightarrow{O_7M}}{dt}\right|_{S} ; \left.\frac{d\overrightarrow{O_7M}}{dt}\right|_{S} = \left.\frac{d}{dt}(e\overrightarrow{x_7} + R\overrightarrow{y_7})\right|_{S} = \vec{0}$$

$$\left.\frac{d\overrightarrow{O_0M}}{dt}\right|_{R_0} = \left.\frac{d\overrightarrow{O_0O_5}}{dt}\right|_{R_0} + \left.\frac{d\overrightarrow{O_5O_6}}{dt}\right|_{R_0} + \left.\frac{d\overrightarrow{O_6O_7}}{dt}\right|_{R_0} + \left.\frac{d\overrightarrow{O_7M}}{dt}\right|_{R_0}$$

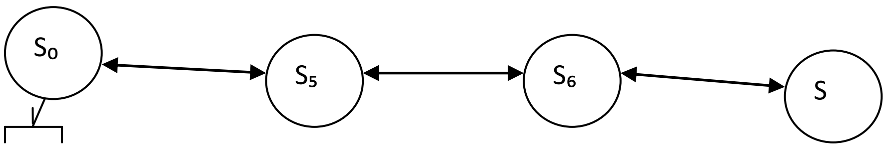
$$\Rightarrow \overrightarrow{V_{M\in S/R_0}} = \dot{\mu}\overrightarrow{x_0} - R\dot{\theta}_{76}\sin\theta_{76}\overrightarrow{y_0} + (\dot{\delta} + R\dot{\theta}_{76}\cos\theta_{76})\overrightarrow{z_0}$$

1.2.3

$$\overrightarrow{\Gamma_{M/R_0}} = \left.\frac{d\overrightarrow{V_{M\in S/R_0}}}{dt}\right|_{R_0}$$ Facile à calculer.

3. *Etude d'hyper statisme :*

h-mc=Ec-Ic



$$\begin{Bmatrix} \overrightarrow{0} \\ \dot{\mu}\overrightarrow{x_0} \end{Bmatrix}_{O_5=O_7} \qquad \begin{Bmatrix} \overrightarrow{0} \\ \dot{\delta}\overrightarrow{z_0} \end{Bmatrix}_{O_6=O_7} \qquad \begin{Bmatrix} \dot{\theta}_{76}\overrightarrow{x_0} \\ \vec{0} \end{Bmatrix}_{O_7}$$

$$\{\vartheta_{S/0}\}_{O_7} = \begin{Bmatrix} \dot{\theta}_{76}\overrightarrow{x_0} \\ \dot{\mu}\overrightarrow{x_0} + \dot{\delta}\overrightarrow{z_0} \end{Bmatrix}_{O_7}$$ $\Rightarrow$ mc=3

H=3+0-3=0 (Rq et vérification : chaine ouverte =>H=0)

4. *Etude de la résistance des matériaux :*

$$\{\tau_{S_7\to S_8}\}_I = -\{T\}_I$$ Et $$\overrightarrow{M}_I = \overrightarrow{M}_J + \overrightarrow{IJ} \wedge \vec{R} = \begin{pmatrix} 10^6 \\ -19900 \\ -39850 \end{pmatrix}$$

Donc $$\{\tau_{S_7\to S_8}\}_I = \left\{\begin{array}{c|c} -100 & -10^6 \\ 200 & 19900 \\ -100 & 39850 \end{array}\right\}_I$$

1.4.2

$\{\tau_{coh}\}_G = \{T\}_G$ Et on a $\overrightarrow{M}_G = \overrightarrow{M}_J + \overrightarrow{GJ} \wedge \overrightarrow{R} = \begin{pmatrix} 10^6 \\ 100 \\ 150 \end{pmatrix} + \begin{pmatrix} L-x \\ 0 \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 100 \\ -200 \\ 100 \end{pmatrix}$

Donc $\{\tau_{coh}\}_G = \left\{ \begin{array}{l|l} 100 & 10^6 \\ -200 & 100(1-L+x) \\ 100 & 150-200(L-x) \end{array} \right\}_G$

1.4.3 Condition de résistance :

$$\tau_{\max} = k.\frac{M_t}{I_0}.V \leq \frac{\tau_e}{s} \qquad \text{Avec} \quad V = \frac{De}{2} \quad \text{et} \quad I_0 = \frac{\pi(De^4 - Di^4)}{32} = \frac{\pi(1-0.7^4)De^4}{32}$$

Et on a : Di=0.7 De, k=1.5, s=3, Mt=10^6

AN : De=68mm ; Di=48mm

II. Étude du système porte pièce :

1) Étude cinétique :

2.1.1

$(\overrightarrow{O_3}, \overrightarrow{y_3})$ Axe de révolution ➔ $[Io_3(\Sigma)] = \begin{pmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{pmatrix}_{(-,\overrightarrow{y_3},-)}$ et $A = \frac{B}{2} + \int y^2 dm$

2.1.2

$$\overrightarrow{V_{O_3\in\Sigma/R_0}} = \left.\frac{d\overrightarrow{HO_3}}{dt}\right|_{R_0} = \dot{\lambda}\overrightarrow{y_1} = \dot{\lambda}(\sin\theta\overrightarrow{x_2} + \cos\theta\overrightarrow{y_2})$$

$$\overrightarrow{\sigma_{O_3(\Sigma/R_0)}} = \overline{\overline{Io_3(\Sigma)}}.\overrightarrow{\Omega_{\Sigma/R_0}} = \begin{pmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{pmatrix}_{(-,\overrightarrow{y_3},-)} . \begin{pmatrix} 0 \\ \dot{\theta}_{32} \\ \dot{\theta}_{21} \end{pmatrix}_{B_2} = \begin{pmatrix} 0 \\ B.\dot{\theta}_{32} \\ A.\dot{\theta}_{21} \end{pmatrix}_{B_2}$$

$$\{\xi_{\Sigma/R_0}\}_{O_3} = \left\{ \begin{array}{c} m_\Sigma.\overrightarrow{V_{O_3\in\Sigma/R_0}} \\ \overrightarrow{\sigma_{O_3(\Sigma/R_0)}} \end{array} \right\}_{O_3} = \left\{ \begin{array}{l|l} \dot{\lambda}m_\Sigma \sin\theta & 0 \\ \dot{\lambda}m_\Sigma \cos\theta & B.\dot{\theta}_{32} \\ 0 & A.\dot{\theta}_{21} \end{array} \right\}_{(O_3,B_2)}$$

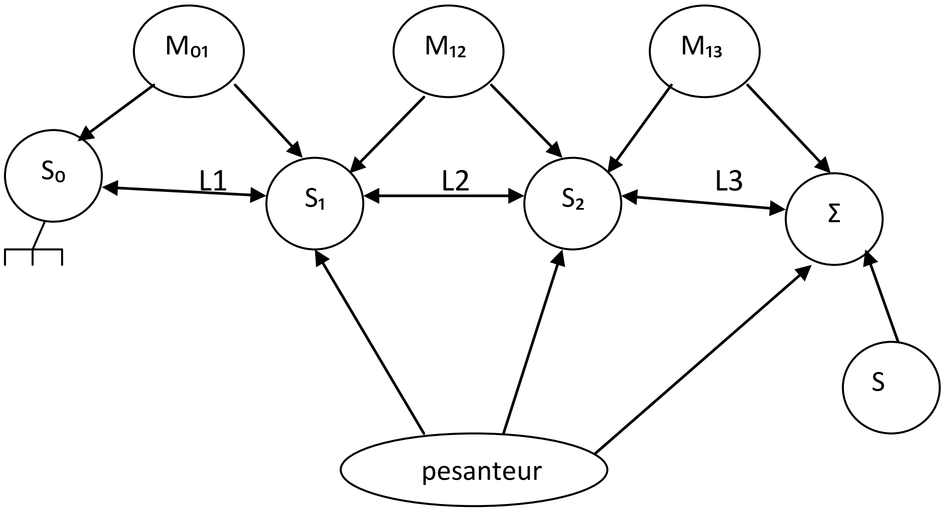
2) ***étude dynamique :***

2.2.1

L1 : Glissière d'axe $(O_1, \overrightarrow{y_1})$

L2 : Pivot d'axe $(O_2, \overrightarrow{z_2})$

L3 : Pivot d'axe $(O_3, \overrightarrow{y_2})$



2.2.2

$$\overrightarrow{\Gamma_{O_3\in\Sigma/R_0}} = \left.\frac{d\overrightarrow{V_{O_3\in\Sigma/R_0}}}{dt}\right|_{R_0} = \ddot{\lambda}\overrightarrow{y_1} = \ddot{\lambda}(\sin\theta_{21}\overrightarrow{x_2} + \cos\theta_{21}\overrightarrow{y_2})$$

$$\overrightarrow{\delta_{O_3(\Sigma/R_0)}} = \left.\frac{d\overrightarrow{\sigma_{O_3(\Sigma/R_0)}}}{dt}\right|_{R_0} = \left.\frac{d\overrightarrow{\sigma_{O_3(\Sigma/R_0)}}}{dt}\right|_{2} + \overrightarrow{\Omega_{2/R_0}} \wedge \overrightarrow{\sigma_{O_3(\Sigma/R_0)}} = \begin{pmatrix} -B.\dot{\theta}_{32}.\dot{\theta}_{21} \\ B.\dot{\theta}_{32} \\ A.\dot{\theta}_{21} \end{pmatrix}_{B_2}$$

$$\left\{\mathfrak{D}_{\Sigma/R_0}\right\}_{O_3} = \begin{Bmatrix} m_\Sigma.\overrightarrow{\Gamma_{O_3\in\Sigma/R_0}} \\ \overrightarrow{\delta_{O_3(\Sigma/R_0)}} \end{Bmatrix}_{O_3} = \left\{\begin{array}{l|l} \ddot{\lambda}m_\Sigma \sin\theta & -B.\dot{\theta}_{32}.\dot{\theta}_{21} \\ \ddot{\lambda}m_\Sigma \cos\theta & B.\dot{\theta}_{32} \\ 0 & A.\dot{\theta}_{21} \end{array}\right\}_{(O_3,B_2)}$$

2.2.3 PFD

III. Etude de Fabrication : (voir cours)